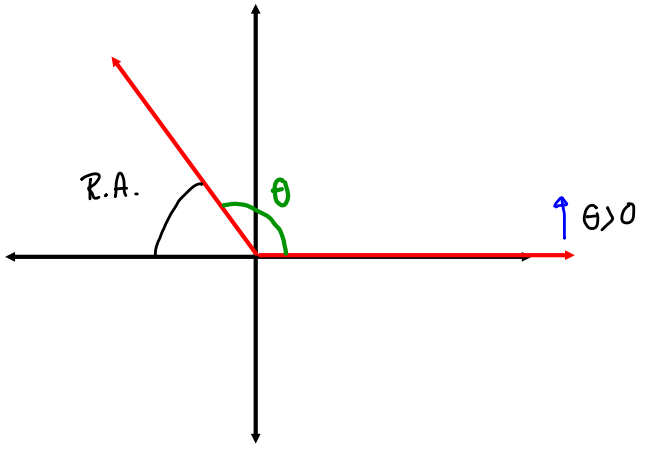
Reference Angle and How to Use Them

Consider angle $\theta > 0$ in standard position

Ref. Angle is an acute angle that the terminal side of angle θ makes with x-axis.



$\theta = 40^\circ$

R.A. $= 40^\circ$

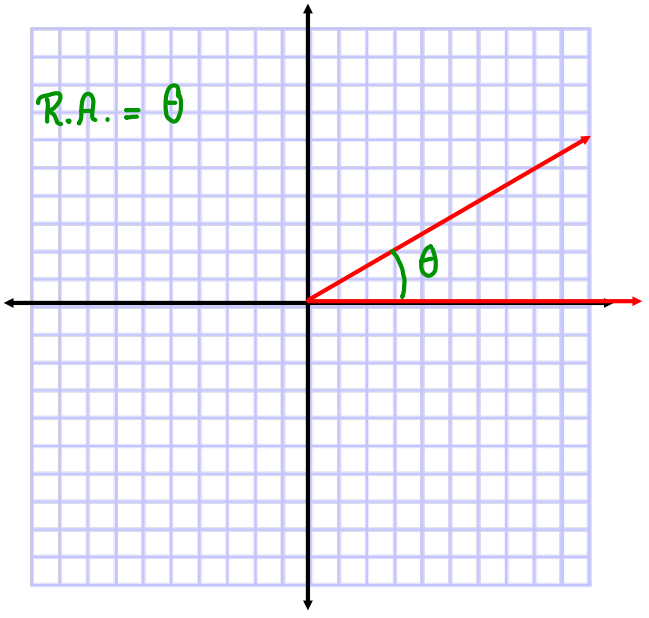
$\theta = \frac{\pi}{3}$

R.A. $= \frac{\pi}{3}$

$\theta = 405^\circ$

$= 360^\circ + 45^\circ$

R.A. $= 45^\circ$



R.A. $= \pi - \theta$

R.A. $= 180^\circ - \theta$

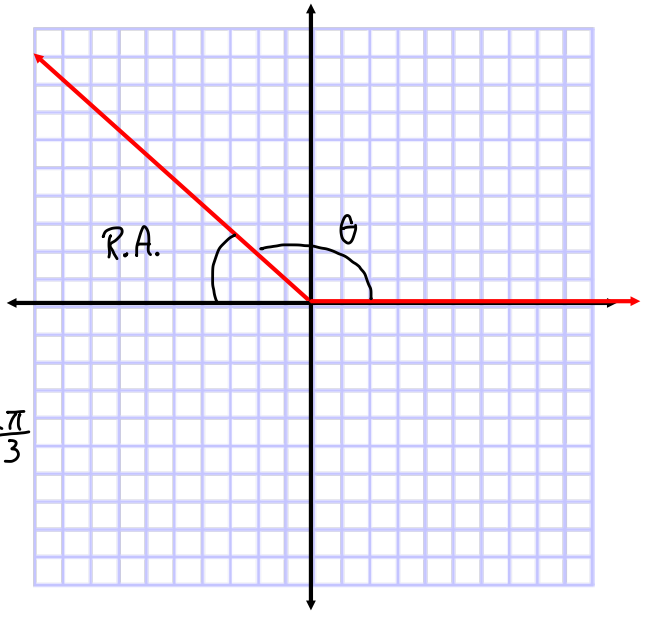
ex: $\theta = 150^\circ$

R.A. $= 180^\circ - 150^\circ = 30^\circ$

$\theta = \frac{2\pi}{3}$

R.A. $= \pi - \frac{2\pi}{3} = \frac{3\pi}{3} - \frac{2\pi}{3}$

$= \frac{\pi}{3}$



$R.A. = \theta - \pi$

$R.A. = \theta - 180^\circ$

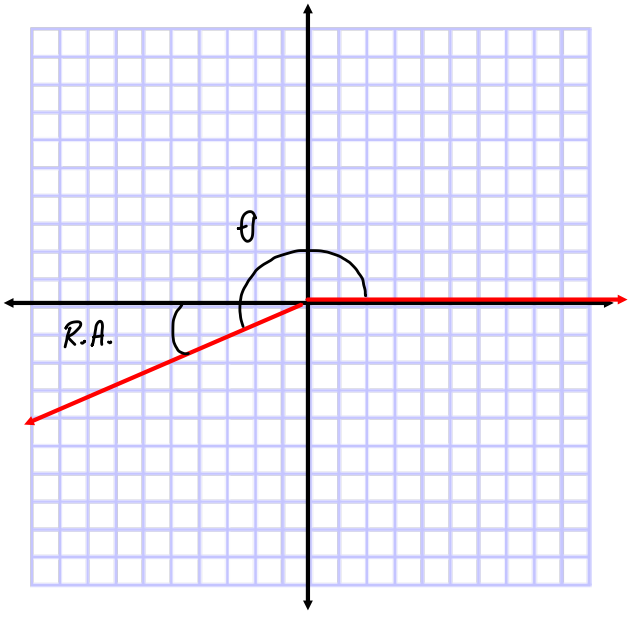
ex: $\theta = \frac{5\pi}{4}$

$R.A. = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$

$\theta = 260^\circ$

$R.A. = 260^\circ - 180^\circ$

$= 80^\circ$



$R.A. = 2\pi - \theta$

$R.A. = 360^\circ - \theta$

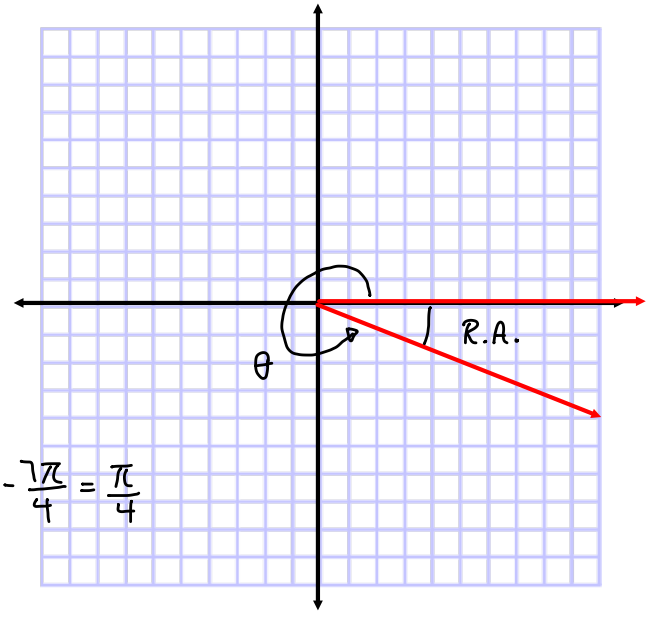
ex: $\theta = 300^\circ$

$R.A. = 360^\circ - 300^\circ$

$= 60^\circ$

$\theta = \frac{7\pi}{4}$

$R.A. = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$



Find reference angle for $\theta = -150^\circ$

1) -150° is in QIII

$-150^\circ + 360^\circ = 210^\circ$

210° is is QIII

R.A. $= 210^\circ - 180^\circ = 30^\circ$

Find the ref. angle for $\theta = -\frac{7\pi}{4}$

1) $-\frac{7\pi}{4}$ is in QI

$-\frac{7\pi}{4} + 2\pi = -\frac{7\pi}{4} + \frac{8\pi}{4} = \frac{\pi}{4}$ $\frac{\pi}{4}$ is in QI.

R.A. $= \frac{\pi}{4}$

Find Ref. Angle for $\theta = 1000^\circ$

$1000^\circ - 360^\circ = 640^\circ$

$640^\circ - 360^\circ = 280^\circ$

280° is in QIV

$R.A. = 360^\circ - 280^\circ$

$= 80^\circ$

Find Ref. angle for $\theta = \frac{-51\pi}{4}$

$-\frac{51\pi}{4} = \frac{-48\pi}{4} + \frac{-3\pi}{4}$

$= -12\pi + \frac{-3\pi}{4}$

$= -6(2\pi) + \frac{-3\pi}{4}$

$R.A. = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$

$\frac{-3\pi}{4} + 2\pi = \frac{-3\pi}{4} + \frac{8\pi}{4} = \frac{5\pi}{4}$

QIII

How to use ref. angle to evaluate trig. functions:

1) Identify the quadrant

2) find ref. angle

3) Value of trig. function is the same as the value of trig. function of ref. angle except possibly the sign.

$\cos 135^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

$\sin \frac{5\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\tan 585^\circ = \tan 225^\circ = \tan 45^\circ = 1$

$\cos\left(-\frac{7\pi}{3}\right) = +\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$